Directions: Read each problem carefully. Remember to show all work in a logical and legible manner. To receive credit, all answers must follow logically from the work that you show on your exam.

1. [4] Consider a rectangle whose area is 432 square feet. The length of the rectangle is 3 times the width. What are the dimensions?

2. [4] Solve using the quadratic formula: $6x^2 + 5 = 40x - 10x^2$

3. [4] Solve for $x$: $\sqrt{x + 1} = 3x + 1$

4. [6] The endpoints of the diameter of a circle are (-5,1) and (7,6).
   a. Find the center of the circle.
   b. Find the radius of the circle.
   c. Find the standard form of the equation of the circle.
5. [5] A salesperson at radio shack receives a monthly salary (before taxes) of $1700 plus a commission of 10% of sales for the month.
   a. If a salesman has \( x \) dollars worth of sales for a month, write an equation for his salary \( S \).

   \[ S = \]

   b. Use the equation in part (a) to find the amount of sales \( x \) the salesperson must have to earn a monthly salary of $2100.

6. [6] State the domains of the following functions in interval notation.
   Example: \( f(x) = \sqrt{x + 2} \) Domain: \([-2, \infty)\)
   a. \( f(x) = 3\log_4(x) \) Domain: 
   
   b. \( f(x) = 3e^{2x} \) Domain: 
   
   c. \( f(x) = \frac{x^3}{x-8} \) Domain: 

7. Consider the following graph: (if none exist, write none)
   a. [3] On what intervals (if any) is the graph increasing? __________

   decreasing? __________

   constant? __________

   b. [2] Is \( y \) a function of \( x \)? \textit{Explain your reasoning.}

   c. [2] Does this graph have an inverse function? \textit{Explain your reasoning.}

   d. [1] What is the domain of this function?

   e. [1] What is the range of this function?

8. [4] Consider the graph of \( f(x) = x^2 - 5x \). Use your knowledge of transformations to write an equation for the following description:

   The graph of \( f \) is shifted four units up, reflected about the \( y \)-axis, and vertically stretched by a factor of 3.
9. [4] Let \( f(x) = 3x - 2 \) and \( g(x) = x^2 + 3 \). Find the following:
   
   a. \( (f - g)(2t) = \) ______________
   
   b. \( (f \circ g)(x) = \) ______________

10. [4] Determine whether \( f(x) = \frac{3x + 4}{5} \) has an inverse function. **Show your work.** If it does, find its inverse.

11. [3] A manufacturer of chairs has daily production costs \( C \) (in dollars per chair) of

   \[ C(x) = 0.3x^2 - 12x + 5400 \]

   where \( x \) is the number of chairs produced. How many chairs should be produced each day to yield a minimum cost per unit?

12. [6] For the function

   \[ g(x) = \frac{x^3 + 2x + 1}{x^2 - 2x - 15} \]

   find the following asymptotes. Write each asymptote as an equation.

   a. All horizontal asymptotes (if any exist)

   b. All vertical asymptotes (if any exist)

   c. All slant asymptotes (if any exist)
13. [4] Consider the following function

\[ f(x) = -2x^9 + 4x^3 - 17 \]

a. What is the maximum number of zeros? ___________________________

b. What is the maximum number of turning points? __________________

c. What is the right-hand behavior? ________________________________

d. What is the left-hand behavior? _________________________________

14. Students in a history class were given an exam. During the next two years, the same students were retested several times. The average score \( g \) can be approximated by the model

\[ g(t) = 87 - 16 \log_{10}(t + 1) \]

where \( 0 \leq t \leq 24 \) and \( t \) is the time in months.

a. [1] What was the average score on the original exam?

b. [1] What was the average score after 6 months?

c. [3] When did the average score drop below 70?

15. [4] Condense the expression \( 2 \log_{10}(t+1) - \log_{10}z + \log_{10}(2) \) as much as possible.

16. [4] Expand the expression \( \log_2 \left( \frac{2x}{z} \right) \) as much as possible.

17. [4] Solve the following for \( x \):

\[ \ln x - \ln(x + 1) = 3 \]
18. [5] A college graduate invests $15,000 of her savings at the age of 25 in an account that earns 5.2% compounded monthly. If she allows all of this investment to grow at the given compounded rate and makes her first withdrawal at the age of 50, what will be the balance of her account just before her first withdrawal? Express the answer to the nearest penny. [Show the formula used and the steps you used in the process of solving the problem.]

19. Certain bacteria grow according to the model $y = Ae^{bx}$ where $y$ denotes the number of bacteria after $x$ hours have elapsed. If the initial population of the bacteria is 15,000 and after four hours the population is 25,000, find
   c. [2] the number of bacteria expected after seven hours.

20. [4] Solve for $x$ and $y$ in the following system. If the system does not have a solution or has infinitely many solutions indicate so.

   $\frac{1}{2}x - 3y = 1$
   $x + 6y = 0$
21. [5] Solve the following system of equations. If the system does not have a solution or has infinitely many solutions indicate so.

\begin{align*}
    x - y + 3z &= 1 \\
    2y - z &= 2 \\
    -3y + 2z &= 0
\end{align*}

Bonus: [5] John Sullivan and Chuck Norris get into a circular wrestling cage. If \( x \) is the number of times John runs around the cage, and \( y \) is the number of roundhouse kicks Chuck delivers to John, how quickly will John hit the ground when dropped from a 30 foot building?