138 Exam 3 Practice Solutions

1. Find all the real zeros of the function algebraically. Be sure to include the multiplicity of repeated zeros.
   \[ f(x) = x^4 - x^3 - 20x^2 \]
   \[ f(x) = x^2(x^2 - x - 20) = x^2(x - 5)(x + 4) \]
   The zeros are:
   \( x = 0 \) which has multiplicity 2
   \( x = 5 \)
   \( x = -4 \)

2. Describe the right-hand and left-hand behavior of the graph of the polynomial function.
   \[ g(x) = 6 - 4x^2 + x - 3x^5 \]
   Degree: 5 (odd) Leading Coefficient: \(-3\) (negative)
   Left Behavior: As \( x \to -\infty \), \( g(x) \to \infty \)
   Right Behavior: As \( x \to \infty \), \( g(x) \to -\infty \)

3. How many turning points and zeros can the following function have?
   \[ g(x) = 6 - 4x^2 + x - 3x^5 \]
   Degree: 5 Maximum number of zeros: 5 maximum number of turning points: 4 \((5-1)\)

4. Use long division to divide \( 6x^3 + 10x^2 + x + 8 \) by \( 2x^2 + 1 \).
   \[ 3x + 5 + \frac{-2x + 3}{2x^2 + 1} \]
   (which is equivalent to \( 3x + 5 - \frac{2x-3}{2x^2+1} \))

5. Use synthetic division to divide \( 6x^3 + 10x^2 + x + 8 \) by \( x - 2 \).
   \[ 6x^2 + 22x + 45 + \frac{98}{x-2} \]

6. Let \( f(x) = \frac{x^2}{x^2 - 4} \). Find the following (if they do not occur, write "none").
   a. \( x \)-intercept(s): \((0,0)\)  
   b. \( y \)-intercept: \((0,0)\)  
   c. horizontal asymptote(s): \( y = 1 \)
   d. vertical asymptote(s): \( x = -2 \) and \( x = 2 \)  
   e. slant asymptote(s): none
   f. Sketch the graph of \( f \):
7. Let \( f(x) = \frac{x^3}{2x^2 - 4} \). Find all asymptotes.

Vertical: \( x = \sqrt{2} \) and \( x = -\sqrt{2} \)  

Horizontal: none (larger degree in the numerator)

Slant: degree of the numerator is EXACTLY one more than the degree of the denominator so this will have a slant asymptote. Use polynomial division to find it.

\[
x^3 \div 2x^2 - 4 = \frac{1}{2} x + \frac{2x}{2x^2 - 4}
\]

Thus the slant asymptote is \( y = \frac{1}{2} x \)

8. Find a degree 4 polynomial that has a zero at \( x = 3 \), a zero at \( x = 1 \), a multiplicity two zero at \( x = -2 \), and passes through the point \((0,4)\).

\[
f(x) = \frac{1}{3} (x - 3)(x - 1)(x + 2)^2
\]

9. Match the following functions with their domains. Fill in the blank with the correct letter (a-k) from the lists on the left. Not all letters will be used and some may be used more than once.

\[
\begin{align*}
\text{e} & \quad f(x) = \ln(x - 3) \\
\text{f} & \quad f(x) = \sqrt{x - 3} \\
\text{d} & \quad f(x) = 3^{-x} \\
\text{b} & \quad f(x) = \log_3 x \\
\text{k} & \quad f(x) = \frac{3x^2}{x - 3}
\end{align*}
\]

\[
\begin{align*}
\text{a} & \quad [-3, 3] \\
\text{b} & \quad (0, \infty) \\
\text{c} & \quad [0, \infty) \\
\text{d} & \quad (-\infty, \infty) \\
\text{e} & \quad (3, \infty) \\
\text{f} & \quad [3, \infty) \\
\text{g} & \quad (-3, 3) \\
\text{h} & \quad (-3, \infty) \\
\text{i} & \quad (-\infty, -3) \\
\text{j} & \quad (-\infty, -3)
\end{align*}
\]
10. One investment paid 8% compounded annually and another paid 7.5% compounded continuously. If $4,000 was invested in both accounts for 3 years, which is the better investment?

   Balance in 8% account = $5038.85

   Balance in 5% account = $5009.29

   The 8% is a better investment.

11. Convert the equation $10^{-3} = 0.001$ to a logarithmic equation.

   \[ \log_{10} 0.001 = -3 \]

12. Convert the equation \( \ln \left( \frac{1}{e} \right) = -1 \) to an exponential equation.

   \[ e^{-1} = \frac{1}{e} \]

13. Convert the equation \( \log_3 2x = t \) to an exponential equation

   \[ 3^t = 2x \]

14. Let \( f(x) = 3(1.9)^x \) Find the following (if they do not occur, write "none").
   a. x-intercept(s): none
   b. y-intercept(s): (0,3)
   c. horizontal asymptote(s): \( y = 0 \)
   d. vertical asymptote(s): none
   e. Domain: all real numbers
   f. Range: \( (0, \infty) \)
   g. Sketch the graph of \( f \):

15. Solve \( 170 = \frac{500}{1+4e^{-2k}} \) for \( k \).

   \[ k = \frac{\ln \left( \frac{33}{38} \right)}{-2} \approx 0.362 \]
16. Let \( f(x) = \log_3(x + 2) \). Find the following (if they do not occur, write "none").

a. x-intercept(s): \((-1, 0)\)
b. horizontal asymptote(s): none
c. vertical asymptote(s): \(x = -2\)
d. Domain: \((-2, \infty)\)
e. Range: all real numbers
f. Sketch the graph of \( f \):

17. Deedra has $1,800 which she wants to grow to $2,500 in three years. What continuously compounded interest rate does she need to get in order to achieve her goal?

\[ r = 10.95\% \]

18. If after \( w \) weeks of training, Shelby rides a horse at \( S = 60(1 - e^{-0.08w}) \) km per hour, how many weeks will it take her to ride at 20 km per hour?

\[ w = 5.068, \text{ or about } 5 \text{ weeks} \]

19. On the day a child was born, a lump sum was deposited in a trust fund paying 6.5% interest compounded continuously. How much money should have been deposited so that the trust will be worth 1,000,000 on the child's 25th birthday?

\[ P = $196,911.68 \]

20. If Bahkia puts 35 ants into her ant farm and has 85 ants 3 months later:
   a. How many ants will she have after 1 year? \( A = 1218 \) (and \( r = 0.2958 \))
   b. When will she have 1000 ants? \( t = 11.33 \text{ or } 11.3 \text{ months} \)