Mth 512. Assignment 11.
51 points
(5 bonus points for submitting your solutions in LATEX.)

Your Name

Spring 2015

1. (5 points) Prove that $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ are not isomorphic as fields.

Solution:

2. (5 points) Let $\mathbb{F}$ be a finite field of characteristic $p$. Prove that $|\mathbb{F}| = p^n$ for some positive integer $n$.

Solution:

3. Consider the field extension $\mathbb{Q}(\sqrt[4]{3},i)$ over $\mathbb{Q}$.

   (a) (5 points) Find a basis for the field extension $\mathbb{Q}(\sqrt[4]{3},i)$ over $\mathbb{Q}$. Conclude that $[\mathbb{Q}(\sqrt[4]{3},i) : \mathbb{Q}] = 8$.

Solution:

(b) (5 points) Find all subfields $F$ of $\mathbb{Q}(\sqrt[4]{3},i)$ such that $[F : \mathbb{Q}] = 2$.

Solution:

(c) (5 points) Find all subfields $F$ of $\mathbb{Q}(\sqrt[4]{3},i)$ such that $[F : \mathbb{Q}] = 4$.

Solution:

(d) (5 points) Draw a lattice diagram for all of the subfields of $\mathbb{Q}(\sqrt[4]{3},i)$ over $\mathbb{Q}$. (5 bonus points if you can do it in TIKZ.)
4. (3 points) Determine the degree of the extension $\mathbb{Q}(\sqrt{3 + 2\sqrt{2}})$ over $\mathbb{Q}$.

Solution:

5. (5 points) Prove that if $[F(\alpha) : F]$ is odd then $F(\alpha) = F(\alpha^2)$.

Solution:

6. (3 points) Prove that it is impossible to construct the regular 9-gon.

Solution:

7. (5 points) Determine the splitting field and its degree over $\mathbb{Q}$ for $x^4 - 2$.

Solution:

8. (5 points) Determine the splitting field and its degree over $\mathbb{Q}$ for $x^4 + 2$.

Solution: