The William Lowell Putnam Mathematical Competition

September 12, 2014

The William Lowell Putnam Mathematical Competition is an annual contest for college students established in 1938 in memory of its namesake. Cash prizes for the top five teams in recent years ranged from $25,000 to $5,000. Recent cash prizes for the top five individuals have been $2,500 each. Below the individual winners for each year are listed in alphabetical order. The Elizabeth Lowell Putnam Prize was established in 1992 to be "awarded periodically to a woman whose performance on the Competition has been deemed particularly meritorious". Recent winners of the Elizabeth Lowell Putnam prize have received $1,000. Over the years many of the winners of the Putnam competition have become distinguished mathematicians. A number of them have received the Fields Medal and several have won the Nobel Prize in Physics.

The Annual William Lowell Putnam Mathematical Competition will be on the first Saturday in December. The exam is divided into two sessions. Participants take the first part of the exam in the morning (9:00 am - 12:00 pm) and the second part in the afternoon (2:00 pm - 5:00 pm). You must sign up by October 1, 2014. For more information, see Dr. Beavers or Dr. Judson.

Practice Problem of the Week

Each Friday two or three practice problems will be posted online at

http://faculty.sfasu.edu/judsontw/putnam/index.html

as well as solutions for problems from the previous week. The problems for this week are:

1. Show that among any $n + 1$ integers, there must be two integers whose difference is a multiple of $n$.

   **Solution:** The two desired numbers must have the same remainder upon division by $n$. Since there are only $n$ possible remainders, we are done by the pigeonhole principle.

2. Let

   \[ f(x) = \prod_{k=0}^{n} (x + k) = x(x + 1)(x + 2) \cdots (x + n). \]
Find $f'(1)$.

**Solution:** First note that

$$\ln(f(x)) = \ln(x) + \ln(x + 1) + \cdots + \ln(x + n).$$

Differentiating, we have

$$\frac{f'(x)}{f(x)} = \frac{1}{x} + \frac{1}{x + 1} + \cdots + \frac{1}{x + n}.$$ 

Thus,

$$f'(1) = (n + 1)! \left(1 + \frac{1}{2} + \cdots + \frac{1}{n + 1}\right).$$