The William Lowell Putnam Mathematical Competition

November 14, 2014

1. Prove that for each positive integer $n$, there are pairwise relatively prime integers $k_0, k_1, \ldots, k_n$, all strictly greater than 1, such that $k_0k_1\cdots k_n - 1$ is the product of two consecutive integers.

2. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a function such that $f(x, y) + f(y, z) + f(z, x) = 0$ for all real numbers $x, y, z$. Prove that there exists a function $g : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x, y) = g(x) - g(y)$ for all real numbers $x$ and $y$. 